The design of the Snefru pyramids at Dahshur and the Netjerikhet pyramid at Saqqara

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ISSN 1567-214X

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Abstract

This investigation concerns the mathematical structure in the design of the north and south pyramids at Dahshur. The odd form of the Bent pyramid and the architectural layout of the two pyramids will be ascribed to an organic theory of the emerging solar religion, in a logical parallelism with the Great pyramid, and in the context of principals of sacred mathematics that in some aspects can be traced back to the plan of the Step pyramid at Saqqara.

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1. The Bent pyramid

The Bent pyramid is sometimes simply considered a makeshift solution, in consideration of the constructional problems the builders experimented, but the analysis of its structure reveals a careful planning and an organic conception (table 1).

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>Side of base I</td>
<td>361.84</td>
<td>360.18</td>
<td>362</td>
<td>362 = 9 x 40 + 2</td>
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<tr>
<td>Side of base II</td>
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<tr>
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<td>54°31’</td>
<td>55°</td>
<td>55°</td>
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<tr>
<td>Upper height</td>
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<td>106.95</td>
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<tr>
<td>Bending line I</td>
<td></td>
<td></td>
<td>236</td>
<td>236 = 9 x 26 + 2</td>
</tr>
<tr>
<td>Bending line II</td>
<td></td>
<td></td>
<td>234</td>
<td>234 = 9 x 26</td>
</tr>
<tr>
<td>Upper slope</td>
<td>43°05’</td>
<td>43°21’</td>
<td>43°</td>
<td>43°</td>
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</table>

Table 1. The measures of the Bent pyramid, in royal cubits (1 cubit = 0.5236 m) (after Dorner, 1987).

The survey of Dorner (1987) leads to an estimation of 4 + 9/10 palms for the skd (run for a rise of 7 palms) in the lower part of the pyramid (a ratio of 10/7), and 7 + 1/2 palms for the skd in the upper part. According to Dorner, at the base level of the inner pyramid, the side of the base measures 360 cubits, and 10 palms below, at the new base level, 362 cubits. Figure 1, showing the decomposed measures of the Bent pyramid, indicates also a theoretical line of 234 cubits at a distance of 6.5 palms from the bending line.

Figure 1. The measures in the Bent pyramid are multiples of numbers nine and 11.

The study of Dorner (1987) was questioned by Legon (1990), who suggested that the bending line is 234.8 cubits, on the basis of the measures of Petrie for the lower slope. Both Petrie (1888) and Dorner (1987) agree on an angle of 55° near the base of the pyramid, while an angle of 54° 36’ was estimated by Petrie near the bending line. But the lower run was measured by Dorner and Petrie in 63 cubits in the north and 62.5 cubits in...
the east, with a mean angle of about 55° both in the north and in the east (Dorner, 1992), and the hypothesis of an intended combined angle for the lower slope is not realistic. It should be considered, instead, that the building of a gigantic true pyramid with simple tools and techniques was subject to a margin of error, and that even with small distortions in some points, the precision of the result is impressive.

In the Khufu pyramid the side of the base was 440 = 11 x 40 cubits and the height 280 = 7 x 40 cubits; in the Bent pyramid the side of the base at the second base level was 360 = 9 x 40 cubits and the height 200 = 9 x 10 + 11 x 10 cubits. Petrie (1888) assumed that in the Bent pyramid the intended measure for the side of the base was just 360 cubits. The following analysis suggests that a conceptual model of the Bent pyramid that envisaged a side of the base of 360 cubits and a bending line of 234 cubits was slightly adjusted (to 362 and 236 cubits), with the lowering of the pyramid’s base, in consideration of the levels of the chambers.

The multiplication of the lower height by 11/9 gives the approximate distance of the lower slope (11 x 10 cubits). In the Khufu pyramid, the designers had a predilection for the numbers seven and 11, and in the Bent pyramid for the numbers nine and 11. The number nine was associated to the psd’t of Heliopolis and to the diameter of the circle, and the sacredness of the number seven is attested by several religious references (e.g. the seven palms of the royal cubit, the seven petals/rays of the goddess of measurement Seshat, the seven Hathor, the use of the number seven in ancient litanies, as in Pyramid Texts, utt. 318, etc.).

The first plan of the pyramid was different: the lower courses were built on a narrower base and with a steeper slope (a side of the base of 300 cubits and a skd of 4 + 1/2 palms, according to Dorner (1987)). Some scholars (e.g. Verner, 2001) argued that the further reduction of slope of the enlarged pyramid that resulted in the bend was decided to face structural instability (the core of the lower part was built with the stones sloping inward, while the stones of the upper part were placed horizontally), but others believe it was set in the original second plan. The present analysis supports the latter hypothesis, on the basis of two considerations:

- the design of the pyramid appears to be based on an organic mathematical structure, with religious implications.
- the analysis of the measures suggests that the bend, the two chambers, the chimney, and other architectonic elements, were built to satisfy an organic religious theory.

In figure 2, a semicircle with the bending line as diameter intersects the points of bend and the chimney at the level of the chamber’s floor; a semicircle with about the same diameter intersects the points of bend and the apex of the pyramid. A circle of diameter 9 x 22 intersects the apex of the pyramid and the floor level of the upper chamber. The levels of the chambers would have been determined likewise in the first plan (possible bending line 207 = 9 x 23 cubits at the height of about 73 cubits, with a base of 301 = 7 x 43 cubits), and the effort to take into consideration given levels would explain the slight adjustment of the bending line from 234 = 9 x 26 to 236 cubits and the lowering of the pyramid’s base.

As in the Khufu pyramid, there are two chambers: a lower chamber vertically aligned to the apex of the pyramid (in the case of the Bent pyramid, with an element of the chamber, the chimney), and an upper chamber located south of the lower chamber. The upper burial chambers of Khufu and Snefru lie both above ground level. The interpretation of the two pyramids is similar: the lower chamber represented the sun regenerated in the horizon, while in the upper burial chamber took place the regeneration of the king’s bi-soul and the beginning of its journey to the horizon and the zenith. The exit of the bi from the pyramid to the northern sky at dawn represented the emerging of the sun from the dw’lt in the eastern side of the sky (the ṣḥ.t in which the king united with the sun as a living ḥkh-spirit), and the pyramidion was symbolic of the union with the sun in the zenith (Ra, the phoenix of Heliopolis).

The Great pyramid was called ‘ḥkhet Khufu’, and the division in two parts of the Bent pyramid appears to be inspired by the same effort to celebrate the sun rising in the horizon. This is a central theme of the Pyramid Texts: “The concept of the king’s journey from death to new life enshrined in Unas’ Pyramid Texts parallels that of the sun: dying in the west, uniting with Osiris in the Duat, and rising again in the East” (Allen, 1994: 25). In the ancient texts, the journey of Ra is discontinuous: in the horizon he transfers from the night-bark to the day-bark.

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1 In the Snefru pyramid at Maidum, the side of the base was 275 c = 11 x 25 and the height 175 c = 7 x 25.
2 Any collection of deities was a psd’t. There was a psd’t of seven gods in Abydos.
3 In the Rhind papyrus, the area of a circle of diameter  d = 9 is (d - d/9) x (d - d/9).
4 The chimney’s ceiling is reported at the level of the pyramid’s base, 15.3 m (29 cubit) from the chamber’s floor (Fakhry, 1959; Stadelmann, 1985).
5 Assuming a theoretical bending line of 234c=9x26, the diameter of the semicircle would be 234.44 cubits, with its centre about seven cubit below the bending line. The resulting circle is slightly bent, like the pyramid is bent.
6 For a study of the architectonic symbolism of the later inscribed pyramids, in which the king’s resurrection is paralleled to the journey of the sun see Allen (1994).
The emerging concepts of the solar religion were harmonised with the tradition, as attested by the two separate entrances: the first entrance is in the usual north side and, at the end of a descending corridor with antechamber, leads to the lower corbelled chamber; the second entrance is in the west side and, at the end of a descending corridor with two portcullis barriers, leads to the upper corbelled chamber.

Figure 2. Interpretation of the religious meaning of the Bent pyramid.

The Bent pyramid was called $h^j-r.sy$-snfrw, Snefru shines - south (pyramid). The upper chamber is located south-east of the lower chamber (figure 3). Substituting the east-west orientation for the traditional north-south orientation, in the described ideal model of the resurrection the king would shine like the sun in the south of the pyramid.

Figure 3. Section of the Bent pyramid from above.
Another peculiarity of the pyramid is the ‘chimney’: an upward shaft vertically aligned to the apex of the pyramid, with a smoothly finished ceiling. An opening and a corbelled niche in the south wall of the lower chamber lead to the chimney. The chimney continues deeper than the chamber’s floor (figure 4’), and there are in fact two upward shafts. In 1948, the lower shaft was cleaned from rubble for about eight metres, but the bottom was not reached; it was refilled (Fakhry, 1959).

In the present interpretation, the upward shaft is a form of the ḏḏ and represented the supporting forces of the sun in the n’dīr. The ḏḏ-pillar symbolised the stable vital strength (backbone) of the resurrecting Osiris and the sun in its rebirth. According to Ricke (1944), Osiris was probably incorporated into the worship of the dead king during the transition from the 3rd to the 4th dynasty.

![Figure 4 (left). Three-dimensional representation of the lower chamber and chimney.](image1)

![Figure 5 (right). The corbelled chimney.](image2)

In Pyramid Texts, utterance 321, the king ascends to Ra in the sky on the backbone of Osiris. The ritual called ‘raising of the ḏḏ pillar’, that was performed in later dynasties, is an indication that the concept of ḏḏ was in relation with a conceptual model in which a directional force determines the circular path of the sun, like the hand of a clock. Probably the niche contained a ḳꜣ-statue of the king, and food offerings actuated the union of the king with Osiris and the conceived sun, the beginning of the ascension of the ḳꜣ to the eastern side of the sky and to Ra, and the regeneration of the bi-soul in the sarcophagus. The offer is described in Pyramid Texts, utt. 321: “O you whose back is on his back, bring to the king the sfrt of the offering-meal which is on the back of Osiris, so that the king may ascend on it to heaven, so that the king may escort Ra in the sky.”

A schematic representation of the top section of the ḏḏ is observable in the corbelled chimney (figure 5’), with four courses that project towards east in the upper part, and four courses that project towards west in correspondence with the niche. The depiction would be split into two parts to indicate the presence of the king beside Osiris. The ḏḏ was a combination of four papyrus reeds associated to the four directions: “Gods of the west, gods of the east, gods of the south, gods of the north! These four pure reed floats which you placed for Osiris when he ascended to the sky, so that he might cross towards the cool region, while his son Horus was beside him so that he might bring him up and let him appear as a great god in the cool region, place them for the king!” (Pyramid Texts, utt. 303).

The four vertebrae in the top section of the ḏḏ were associated to the gods of the four directions, the four children of Horus that raised up and guided to heaven the king (Pyramid Texts, utt. 545) by means of the ‘reed-

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7 Three-dimensional elaboration on the basis of the figures in Fakhry (1959).
8 The corbelled chimney is outlined in Fakhry (1959). Fakhry reports the discovery in the debris around the pyramid of a small copper ḏḏ-amulet of an uncertain date.
9 The ḏḏ depicts “a column imitating a bundle of stalks tied together” (Gardiner, 1927: 502).
10 In a scene from the Temple of Hathor at Denderah, four hawks are perched on four papyrus reeds, which are replaced with the ḏḏ pillar in another scene (Mariette, 1870-1874: Pl. IV: 65, 71).
floats’, and that provided the deceased with the sustenance necessary to its regeneration: “Hapy, Duamutef, Kebhseuf, and Imsety will expel this hunger which is in my belly and this thirst which is on my lips.” (Pyramid Texts, utt. 338).

In conclusion, the beginning of the 4th dynasty was indeed a period of great changes in the religious system, and the effort to reconcile with the tradition the theories of the emerging solar doctrine is evident in the analysis of the Bent pyramid. The odd shape of the pyramid was an expression of sacred mathematics, and the architectural layout was functional to the solar theory of the resurrection.

2. The Red pyramid

The plan of the Red pyramid at Dahshur appears informed by the same principals (table 2). With a side of the base of 420 cubits (about 220 m), the angle 43°22’ results in the height of 198.35 cubits (about 104 m). The height of 198 cubits is a multiple of nine, while the side of the base is a multiple of seven. Note that 198 cubits is also a multiple of 11. G 3a in Giza is another example of pyramid in which the side of the base is a multiple of seven and the height is a multiple of nine (84 = 7 x 12 cubits; 54 = 9 x 6 cubits). The ratio 198/210 in the Red pyramid corresponds to a $skd$ of $7 + 14/33 = 7 + 1/3 + 1/11$ palms.

| Side of the base | 220 m | 420 cubits = 7 x 60 |
| Slope           | 43°22’ | 43°19’          |
| Height          | 104 m  | 198 cubits = 9 x 22 |

Table 2. Measures in royal cubits of the Red pyramid (after Lehner, 1997)

In figure 6, a circle of diameter 9 x 22 intersects the apex of the pyramid and the lower corbelled chamber, whose centre is exactly vertically aligned with the apex. A circle of diameter 9 x 21 intersects the upper corbelled chamber. The orientation of the upper chamber is uncommonly east-west, similarly to the untraditional east-west orientation of the corridor leading to the upper chamber of the bent pyramid, and would indicate that the king belongs to the sun in the east.

Figure 6. Interpretation of the religious meaning of the Red pyramid.

The two beautiful lower corbelled chambers (so-called ‘antechambers’) are almost identical as to dimension and architectural conception. Similarly to the ‘chimney’ of the Bent pyramid, the alignment of the second ‘antechamber’ to the apex of the pyramid would be symbolic of the $n\ddir$, the lowest point of the hidden space where the sun was conceived in $nwt$.

The layout of the chambers in the Snefru pyramids and in the Khufu pyramid, although characterised by important peculiarities, is marked with the same conception of the pyramid as a machine for the ascent of the king to the sky.
3. Imhotep, the forerunner

The analysis of the step pyramid of Netjerikhet at Saqqara indicates that the legendary architect and high priest Imhotep was the forerunner of principles used in later pyramids. Like the Great pyramid, the measures in royal cubits of the sides of the base are multiple of 11, as shown in table 3. The rectangular base of the pyramid would be made of 399 (19 x 21) theoretical squares of side 11 cubits. The distance of 109.02 m for the side of the base measured by Lauer (1936) in the north-south direction is closer to 208 cubits (+ 11 cm) than to 209 cubits (- 41 cm), but this discrepancy seems to be within reasonable limits, considering that the final form was the result of several adjustments in the masonry, and that the points in the south-east and north-east angles are theoretical, as the casing is missing.

<table>
<thead>
<tr>
<th></th>
<th>Lauer (1936)</th>
<th>Design</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side of the base (E-W)</td>
<td>121 m</td>
<td>231 (120.95 m) = 11 x 21</td>
</tr>
<tr>
<td>Side of the base (S-N)</td>
<td>109 m</td>
<td>209 (109.43 m) = 11 x 19</td>
</tr>
<tr>
<td>Height (present)</td>
<td>58.63 m</td>
<td></td>
</tr>
<tr>
<td>Height (original)</td>
<td>59.90 m</td>
<td>7 horizontal sections</td>
</tr>
</tbody>
</table>

Table 3. Measures of the step pyramid of Netjerikhet (after Lauer, 1936).

It is necessary to clarify that the measure of about 62.5 m (119 cubits) for the height, given by several authors (e.g. Stadelmann, 1985), originated from a misinterpretation of Lauer’s (1936) study. Today, the height is 58.63 m (112 cubits), and Lauer assumed, for a reason of structural homogeneity with the other steps, that originally the last step was about 1.3 m higher. Lauer measured the steps along a slope of 74°, and he assumed for the apex of the pyramid along this slope a probable theoretical distance of 119 cubits, corresponding to a height of 59.9 m.

The slope of the steps ranges from 71° to 76.5°, but it is usually 74° (a skd of two palms) and the slope of the first step is certainly 74°, as the distance of its face from the base to the intersection with its bend measures 20 cubits, and the height of this face is 10.02 m. Slope and height of the first step result in a run of 5.5 cubits.

The pyramid was developed from a square building of base 120 x 120 cubits (project M1), that was enlarged to 136 x 136 cubits (M2), and then to 152 x 136 cubits (M3). In the project P1, the mastaba M3 was transformed in a four-stepped pyramid, and then, in the phases P1’ and P2, the monument was adjusted to its final shape with six steps. In P1, a casing of thickness 5.5 cubits was added to the mastaba M3 on all four sides, resulting in a side of the base of 163 = 152 + 11 cubits in the east-west direction, and 147 = 136 + 11 cubits in the north-south direction. A skd of two palms was adopted for the steps, and the height of 19.25 = 5.5 x 7/2 cubits was calculated for the first step at the intersection with its bend. The choice of a skd of two palms was not accidental: a mathematical model probably inspired Imhotep’s plan of the Step pyramid.

<table>
<thead>
<tr>
<th>Area of circle (square)</th>
<th>Diameter = side+H.e.</th>
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<tr>
<td>64</td>
<td>9/1 = 8 + 1</td>
</tr>
<tr>
<td>16</td>
<td>9/2 = 4 + 1/2</td>
</tr>
<tr>
<td>4</td>
<td>9/4 = 2 + 1/4</td>
</tr>
<tr>
<td>1</td>
<td>9/8 = 1 + 1/8</td>
</tr>
<tr>
<td>1/4</td>
<td>9/16 = 1/2 + 1/16</td>
</tr>
<tr>
<td>1/16</td>
<td>9/32 = 1/4 + 1/32</td>
</tr>
<tr>
<td>1/64</td>
<td>9/64 = 1/8 + 1/64</td>
</tr>
</tbody>
</table>

Table 4. Mathematical model

This model is represented in table 4: the progressive halving of the diameter nine of a circle and of the side eight of a square results in the progressive division by four of their equivalent areas. Squares drawn at progressively halved distances, with a square of side 8 cubits at the base, a second square of side 4 cubits at the distance of 7 cubits, etc., result in a pyramid with a skd of two palms. Such a theoretical pyramid has the same volume as a cone constructed with the circles, and would express the equivalence of the pyramid with the primordial hill (bnbn). Indications that the series of seven elements set forth above were considered in ancient times derive from the analysis of the Hkat measure and its relation with the Horus eye series.

11 Area of a circle of diameter d calculated with the formula of the Rhind papyrus: (d - d/9) x (d - d/9).
Note that in all the three initial stages of construction the sides of the base were multiples of eight (8 x 15 in M1; 8 x 17 in M2; 8 x 17, 8 x 19 in M3), and the model above corroborates Stadelmann’s belief that the projects M1 and M2 envisaged a pyramid with a square basis (Stadelmann, 1985). Perhaps, the several modifications in width and height of the pyramid had to simulate the raising of the hill of Atum from the primeval waters of Nun, represented by the blue-tiled chambers underneath the pyramid.

The numbers 5.5 and 11 were used in these measures of the project P1:
- In the run of the steps slope (5.5 cubits)
- In the casing’s thickness (5.5 cubits)

The enlargement of the pyramid in the phases P1’ and P2 resulted in the final six-stepped pyramid. As pointed out by Lauer (1936: 24), in the three lower steps the distance calculated along the slope of 74° is about 20 cubits (the hypotenuse of a right-angled triangle with sides 5.5, 19.25, 20): “Nous trouvons ainsi que le niveau du sommet du premier gradin (à l’intersection de sa paroi et de son glacis) coïncide sensiblement avec le niveau correspondant à une longueur de 20 coudées portées le long d’une pente de 74°. Depuis ce niveau jusqu’au sommet du gradin suivant on aurait encore porté le long de la pente 20 coudées, et de même pour le troisième gradin. Au quatrième gradin on n’aurait plus porté que 19 coudées, puis 18 au cinquième et 17 au sixième.” In accordance with the model of Lauer, the height in cubits of the steps is shown in table 5.

<table>
<thead>
<tr>
<th>Step</th>
<th>Distance along a slope of 74°</th>
<th>Distance from the base along a slope of 74°</th>
<th>Height of step from the base</th>
<th>Height of step</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>20</td>
<td>20</td>
<td>19.25</td>
<td>19.25</td>
</tr>
<tr>
<td>2</td>
<td>20</td>
<td>40</td>
<td>38.50</td>
<td>19.25</td>
</tr>
<tr>
<td>3</td>
<td>20</td>
<td>60</td>
<td>57.75</td>
<td>19.25</td>
</tr>
<tr>
<td>4</td>
<td>19</td>
<td>79</td>
<td>76</td>
<td>18.25</td>
</tr>
<tr>
<td>5</td>
<td>18</td>
<td>97</td>
<td>93.30</td>
<td>17.30</td>
</tr>
<tr>
<td>6</td>
<td>17</td>
<td>114</td>
<td>109.65</td>
<td>16.35</td>
</tr>
</tbody>
</table>

Table 5. The height in cubits of the steps in the pyramid of Netjerikhet at Saqqara.

What is the reason of the progressive reduction in the height of the upper steps? Probably a reason of sacred mathematics. In P1, the probable intended height of the four-stepped pyramid corresponded to a distance of 84 = 7 x 12 cubits along a slope of 74°; in the six-stepped pyramid, this distance was 119 = 7 x 17 cubits.

In the project P2, both sides of the base and sides of the steps, at least in the first step, are multiple of 11: with a run of 5.5 cubits, the side of the first step at the intersection with its bend was 198 = 11 x 18 cubits in the north-south direction, and 220 = 11 x 20 cubits in the east-west direction. The theoretical measures of the steps in the project P2 are shown in table 6 and in figure 7 (north-south section).

<table>
<thead>
<tr>
<th>Step</th>
<th>Height of step from the base</th>
<th>Height of step</th>
<th>Side of step south-north</th>
<th>Side of step east-west</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base</td>
<td>0</td>
<td>0</td>
<td>209 = 11 x 19</td>
<td>231 = 11 x 21</td>
</tr>
<tr>
<td>1</td>
<td>19.25</td>
<td>19.25</td>
<td>198 = 11 x 18</td>
<td>220 = 11 x 20</td>
</tr>
<tr>
<td>2</td>
<td>38.50</td>
<td>19.25</td>
<td>165 = 11 x 15</td>
<td>187 = 11 x 17</td>
</tr>
<tr>
<td>3</td>
<td>57.75</td>
<td>19.25</td>
<td>132 = 11 x 12</td>
<td>154 = 11 x 14</td>
</tr>
<tr>
<td>4</td>
<td>76</td>
<td>18.25</td>
<td>100 = 11 x 9 + 1</td>
<td>122 = 11 x 11 + 1</td>
</tr>
<tr>
<td>5</td>
<td>93.30</td>
<td>17.30</td>
<td>68 = 11 x 6 + 2</td>
<td>90 = 11 x 8 + 2</td>
</tr>
<tr>
<td>6</td>
<td>109.65</td>
<td>16.35</td>
<td>36 = 11 x 3 + 3</td>
<td>58 = 11 x 5 + 3</td>
</tr>
</tbody>
</table>

Table 6. The theoretical measures of the steps in the project P2.

A theoretical portion of 11 cubits is assumed for the accretion of the steps (figure 7). This portion, that ranges from about 12 cubits in the first step to 10-11 cubits in other steps, is 11 cubits in most steps and on average.

12 The measures are in accordance with other studies (e.g. Lehner, 1997).
13 The core of each step is made of two tranches of stonework, whose total thickness is 10-11 cubits (Lauer, 1936).
Two groups of casing blocks belonging to the upper part of the steps were found around the pyramid: the average angle of the stones in the first group is $128^\circ$; the stones in the second group, with an angle of $136^\circ-138^\circ$, probably belonged to the last step. As the accretion layers in the core formed an angle of about $16^\circ$ with the horizontal axis, the casing stones in the upper part of the steps sloped about $22^\circ$ up to the 5th step, and about $31^\circ$ in the 6th step (Lauer, 1936).

![Figure 7. South-north section of the Step pyramid, with theoretical distances.](image)

In the model of Lauer, the upper part of the steps measures 5 cubits along a slope of $74^\circ$, which gives a vertical distance of 4.8 cubits and, assuming a *tranche* of 11 cubits for the accretion of the steps, a slope of $21^\circ12' = \text{atan}(4.8/12.37)$ up to the 5th step. It is also inferable how the angles of $128^\circ$ and $21-22^\circ$ for the casing stones were planned. By means of a simple ruler, it is possible to find that the slope corresponding to a *skd* of nine palms (about $38^\circ$) exceeds the slope corresponding to a *skd* of 18 palms ($21^\circ15'$) approximately by the ratio $2/7$, or $16^\circ$, that is just the inclination on the horizontal axis of the masonry (see figure 8). Note the use of number nine. Several pyramids will be later designed with a figure multiple of 11 for the side of the base: the Snefru pyramid at Maidum ($275 = 11 \times 25$ cubits); the Great pyramid ($440 = 11 \times 40$ cubits); the Unas pyramid ($110 = 11 \times 10$ cubits).

Six steps outline seven horizontal sections from the base to the apex of the pyramid: a further indication that a model in connection with the Horus eye series (see table 4) inspired Imhotep’s plan of the Step pyramid. In the project P1’, with base-side $200 = 8 \times 25$ cubits in the south-north direction and $225 = 9 \times 25$ cubits in the east-west direction, there is both a reference to the numbers nine and eight, that represented respectively the diameter of a circle and the side of a square (see table 4). The pyramid reaches a substantial balance of distances and form in the final project P2, with base-side $208 = 209 = 11 \times 19$ cubits in the north-south direction, $231 = 11 \times 21$ cubits in the east-west direction, and height corresponding to $119 = 7 \times 17$ cubits along a slope of $74^\circ$.

A burial chamber (‘granite vault’), that in the north-south section is vertically aligned to the apex of the pyramid, was built at the bottom of a large shaft that developed up to the roof of the square monument M1. A similar upward shaft connects the underground burial chamber to the roof of a typical mastaba from the 4th dynasty.

The maze of corridors and chambers that branch out around the burial vault has no parallel with later pyramids. This complex substructure was almost exactly replicated outside the pyramid in the south tomb, whose access corridor to the burial chamber shows an east-west orientation, like the corridor that leads to the burial chamber of the Bent pyramid. In the pyramids of Snefru and Khufu, the burial chamber is located south of a chamber aligned to the vertex of the pyramid, and this architecture would have been introduced in the Step pyramid, with a south burial chamber outside the pyramid for the *kꜣ* rather than for the *bꜣ*.14 The whole pyramid

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14 The hypothesis that the south tomb was built for the *kꜣ* was formulated by Ricke (1944) and is upheld by Lehner (1997).
complex, and not only the pyramid, was built to celebrate the power of the king and to bring about his resurrection, and the extraordinary effort to build an impressive substructure for the burial of a statue demonstrates the importance of the concept of $kA$ for the ancient Egyptians.

Figure 8. skd of the masonry and casing stones in the upper part of the steps.

4. Acknowledgements

I thank Dr. C. Rossi for her critical evaluation of the manuscript as well as two anonymous reviewers of previous drafts.

5. Cited literature

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Published: 11 January 2005.
p. 2, 3, 6, 7, 9: skd should be: skd
p. 3: 3ht should be: 3ht
p. 4: ḫi-rsy-snfrw should be: ḫi-rsy-snfrw
p. 7: hkq should be: hkq
p. 9: bβ should be: bβ
p. 9, note 14: kβ should be: kβ
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